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**Linear differential equations of order 1, homogeneous differential equations and the substitution method**

1. Determine the general solutions of the given differential equation. Also, specify an interval where the solution is defined.

1.  $\frac{dy}{dx} = 5y$

2.  $\frac{dy}{dx} + y = e^{3x}$

3.  $y' + 2xy = x^3$

4.  $(x + 4y^2)dy + 2ydx = 0$

5.  $e^x \frac{dy}{dx} = 2x$

6.  $xdy = (x \sin x - y)dx$

7.  $ydx - 4(x + y^6)dy = 0$

8.  $\frac{dy}{dx} + y = \frac{1 - e^{-2x}}{e^x + e^{-x}}$

2. Solve the following linear differential equations with the given initial values.

1.  $\frac{dy}{dx} + 5y = 20$        $y(0) = 2$

2.  $(x + 1)\frac{dy}{dx} + y = \ln x$        $y(1) = 10$

3. Check if the following differential equations are homogeneous. If the equation is homogeneous, determine the homogeneity degree.

1.  $f(x, y) = x^3 + 2xy^2 - y^4/x$

2.  $f(x, y) = \sqrt{x+y}(4x + 3y)$

3.  $f(x, y) = \frac{x^3y - x^2y^2}{(x+8y)^2}$

4.  $f(x, y) = \frac{x}{y^2 + \sqrt{x^4 + y^4}}$

5.  $f(x, y) = \sin \frac{x}{x+y}$

6.  $f(x, y) = \ln x^2 - 2 \ln y$

7.  $f(x, y) = \frac{\ln x^3}{\ln y^3}$

8.  $f(x, y) = (x^{-1} + y^{-1})^2$

9.  $f(x, y) = (x + y + 1)^2$

4. Solve the homogeneous differential equations using the appropriate substitution.

1.  $(x - y)dx + xdy = 0$

2.  $(x + y)dx + xdy = 0$

3.  $xdx + (y - 2x)dy = 0$

4.  $ydx + 2(x + y)dy = 0$

5.  $(y^2 + yx)dx - x^2dy = 0$

6.  $(y^2 + yx)dx + x^2dy = 0$

7.  $\frac{dy}{dx} = \frac{y-x}{y+x}$

8.  $\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$

9.  $y \frac{dy}{dx} = x + 4ye^{-2x/y}$

5. Solve the homogeneous differential equations with the given initial values.

1.  $xy^2 \frac{dy}{dx} = y^3 - x^3$        $y(1) = 2$

2.  $(x + ye^{y/x})dx - xe^{y/x}dy = 0$        $y(1) = 0$

6. Solve the following Bernoulli differential equations using the appropriate substitution.

$$1. x \frac{dy}{dx} + y = \frac{1}{y^2} \quad 2. \frac{dy}{dx} = y(xy^3 - 1) \quad 3. x^2 \frac{dy}{dx} + y^2 = xy$$

7. Solve the following Bernoulli differential equations with the given initial value.

$$1. x^2 \frac{dy}{dx} - 2xy = 3y^4 \quad y(1) = \frac{1}{2}$$

$$2. y^{1/2} \frac{dy}{dx} + y^{3/2} = 1 \quad y(0) = 4$$

8. Solve the following differential equations of the form  $\frac{dy}{dx} = f(Ax + By + C)$ .

$$1. \frac{dy}{dx} = (x + y + 1)^2 \quad 2. \frac{dy}{dx} = \frac{1-x-y}{x+y}$$

$$3. \frac{dy}{dx} = \sin(x + y) \quad 4. \frac{dy}{dx} = 1 + e^{y-x+5}$$

9. Solve the following differential equations (of the form  $\frac{dy}{dx} = f(Ax + By + C)$ ) with the given initial values.

$$1. \frac{dy}{dx} = \cos(x + y) \quad y(0) = \frac{\pi}{4}$$

$$2. \frac{dy}{dx} = \frac{3x+2y}{3x+2y+2} \quad y(-1) = -1$$